Estimation of steep fronted and full wave lightning Channel-Base-Current function parameters using Genetic Algorithm

Praneeth K N, K Chandrasekaran,
Suvith Kumar, Gururaj S Punekar
Department of Electrical and Electronics Engineering,
National Institute of Technology Karnataka,
Surathkal 575025, India.

***Abstract*—Typical negative first-return-stroke (FS) and subsequent-return-stroke (SS) lightning channel-base current (CBC) parameters are evaluated using a Genetic Algorithm (GA). Sum of two Heidler's function parameters are tuned for a typical value of peak current ($I_m \approx 12kA$), and maximum time derivative of current ($(di/dt)_{max} \approx 40kA/\mu s$), for different time to crest (t_m). These parameters are also tuned for median and severe negative FS and SS to accurately obtain full wave of CBC, including characteristics related to wave tail. It is thus shown that GA can be effectively applied for obtaining the lightning current parameters in accordance with data reported in the literature. Further, this approach is used to tune the parameters for steep fronted CBC waveforms. The CBC function parameters thus evolved using GA for severe cases will be useful in lightning related simulation study and research.**

Keywords*—*Channel-base current; Genetic Algorithm; Heidler's function; Lightning; Negative first-return-stroke; Negative subsequent-return-stroke.

I. Introduction

The return stroke phase of lightning discharge is considered to be the major factor responsible for the indirect destructive effects, since most intense electromagnetic radiation occurs during this phase. Based on their characteristics (peak current (I_m) and maximum time derivative of current $(di/dt)_{max}$), lightning return strokes are grouped into first-return-stroke (FS) and subsequent-return-stroke (SS) [1], [2]. The lightning channel current depends on the current at the base of the channel (CBC). These parameters of lightning CBC are available in the literature. They are obtained by measurement, either (i) during natural lightning, or (ii) during artificially generated (triggered) lightning [3], [4]. The process of simulation and analysis of induced lightning electric and magnetic fields (LEMF) in general involves usage of reconstructed current wave-shapes, closely matching those of the field observed wave-shapes. Hence such a study needs an appropriate CBC function for the engineering return stroke models [5], [6]. Based on the field data [7], [8] the wave-front region of the return strokes currents can be characterized by three parameters (I_m, $(di/dt)_{max}$, t_m) for the purpose of modeling. A detailed discussion related to these parameters (and a few more) can be found in a recent publication [9]. Many analytical functions representing lightning CBC have been proposed in the literature [9], [10]. Lightning CBC is best approximated using the mathematical function presented by Heidler et al. [11]. This function has been adopted by International Electrotechnical Commission standard IEC 62305-1 [8] for CBC representation. Sum of two Heidler's functions is commonly used in representing the CBC wave shape of a SS [12], [13], [14], [15].

The Heidler's function has its own set of parameters which are to be identified to achieve the desired wave shape of the CBC. Identifying these parameters is not a straight forward procedure. Heidler et al. have given a graphical method to identify these parameters specific to a wave shape [11]. While adjusting the wave-front of the current waveform, Heidler's function allows nearly independent adjustment of I_m and $(di/dt)_{max}$, but not time to crest t_m. There exists an interdependence between I_m, t_m, and $(di/dt)_{max}$ [19]. To overcome this issue, Javor and Rancic [16] have proposed a new channel base current (NCBC) function. NCBC allows independently adjust I_m, t_m, and $(di/dt)_{max}$. In the process NCBC exhibits a delay characteristic in the wave-front region (as compared to Heidler's function [12]). This delay is not only significant but also comparable with the t_m, particularly for higher values of t_m and $(di/dt)_{max}$ [17]. It is demonstrated [18] that Genetic Algorithm (GA) is a useful tool in determining Heidler's function parameters (related to wave-front only) of the typical values provided in literature, [12], [13] with not attempting to adjust t_m.

In the present study, GA is used to tune the Heidler's function CBC parameters, with the objectives as (i) exploring the possibility of GA application for the wave-front region of a negative SS, with known, typical I_m and $(di/dt)_{max}$ for different time-to-crest t_m (an attempt to adjust all these three independently), (ii) using GA to adjust the full wave characteristics, by including the time to 50% of peak current , t_{50} and the rate of current rise from 30-90% of peak current, t_{30-90} and (iii) evaluating Heidler's Function parameters for a steep-fronted negative FS by adjusting I_m, $(di/dt)_{max}$, t_m and t_{50}.

II. Computation of Heidler's function parameters

A. Genetic Algorithm

A Genetic Algorithm is a general-purpose optimization algorithm based on natural selection, the process accepted as the driving force of biological evolution. GA starts with no knowledge of the correct solution, and depends entirely on evolution operators such as selection, crossover and mutation to arrive at the best solution. In the algorithm, the selection of the natural environment is replaced by artificial selection based on a computed fitness for each design. The term fitness is used as an indicator of the chromosomes goodness; and it is, usually, the objective function of the optimization problem. In GA, strings of numerical values, representing members of the population, replace these chromosomes [20]. As the evolution proceeds through the generations, the fittest offsprings will survive.

GA being a parallel search method, can use more than one initial guess as the starting point. In the present study, the population size is chosen to be 50. The crossover probability is 0.8 and the crossover function chosen is of the type scattered, which generates a random binary vector and chooses the gene from parent A if the vector element is 0 and from parent B otherwise. The mutation probability is 0.001 and the mutation function is of the type adaptive-feasible, which generates an adaptive direction and step length depending on the constraints and the previous successful or unsuccessful operation. The convergence criterion is chosen as 100 generations or change in fitness function value between successive iterations is below 10^{-6}, whichever is attained earlier. The search is started with 50 randomly generated population values for each parameter given in (1). The accuracy of GA and convergence greatly depend on the search region for the parameters. This search region can be restricted by specifying appropriate bounds for the parameters. A flowchart explaining the process is given in figure 1.

B. Heidler's CBC Function

It has been reported in the literature that in case of indirect lightning strikes, steep current wave forms result in most intense LEMFs [13], [17]. The steep front is a characteristic of the return stroke phase of the subsequent return strokes. Thus, the modeling of return stroke is essential for lightning electromagnetic field (LEMF) calculation. Among the lightning return stroke models, most widely used one is the engineering model [5], [6]. In this model spatio-temporal distribution of the current along the lightning channel is related to CBC as a product of its delayed version and a decay factor [17]. Heidler et al. proposed a CBC function popularly known as Heidler's function. This function is a combination of power and an exponential function [12]. In this study, sum of two Heidler's functions is used to represent the CBC (1).

$$i(0,t) = \frac{I_{01}}{\eta_1} e^{-t/\tau_{21}} \frac{(t/\tau_{11})^{n_1}}{1+(t/\tau_{11})^{n_1}} + \frac{I_{02}}{\eta_2} e^{-t/\tau_{22}} \frac{(t/\tau_{12})^{n_2}}{1+(t/\tau_{12})^{n_2}}$$

where,

$$\eta_1 = exp(-\frac{\tau_{11}}{\tau_{21}} * \frac{n_1 \tau_{21}}{\tau_{11}}^{1/n_1})$$
$$\eta_2 = exp(-\frac{\tau_{12}}{\tau_{22}} * \frac{n_2 \tau_{22}}{\tau_{12}}^{1/n_2}) \quad (1)$$

Here, I_{01}, I_{02} are constants which control the amplitude of the CBC; τ_{11}, τ_{12} are front time constants; τ_{21}, τ_{22} are the decay time constants; n_1, n_2 are dimensionless constants controlling the steepness, generally varying between 2 and 10; η_1 and η_2 are the amplitude correction factors.

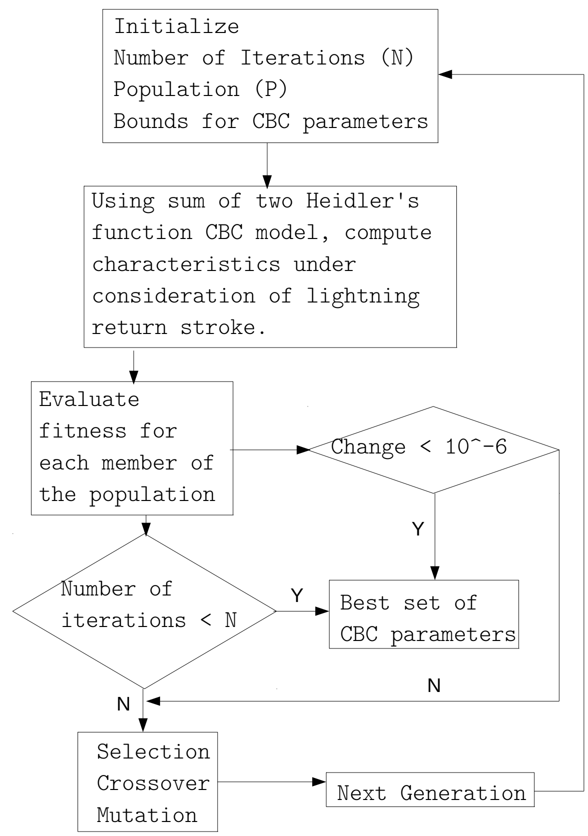

Fig. 1. Flowchart to find Heidler's function parameters to achieve desired CBC characteristics.

C. Tuning wave-front of negative SS

The lightning current parameters for downward negative Cloud-to-Ground lightning are presented by Berger et al.[1]. This data is based on direct current measurements and reported field data . From this database, the typical parameters of negative SS is summarized in Table I. To simulate this typical negative SS using sum of two Heidler's function the appropriate values of function parameters must be computed. Rachidi et al. [13], have identified these parameters

($I_{01},\ I_{02},\ \tau_{11},\ \tau_{21},\ \tau_{12},\ \tau_{22},\ n_1$, and n_2) which are summarized in Table II. In the present study, using GA as the tool, a systematic and independent adjustment of t_m around its typical value (0.8 μs) is attempted.

CBC Heidler's function parameters in (1) constitute the eight variables to be evaluated by GA; to achieve the desired I_m and $(di/dt)_{max}$ corresponding to the SS for different t_m. Hence the objective function (fitness function) value is derived from the computed current peak, maximum time derivative of current, and time to crest of the SS lightning CBC as given in (2).

$$f = \left|\frac{I_{md}-I_{mc}}{I_{md}}\right| + \left|\frac{(di/dt)_{maxd}-(di/dt)_{maxc}}{(di/dt)_{maxd}}\right| + \left|\frac{t_{md}-t_{mc}}{t_{md}}\right| \quad (2)$$

where,
I_{md} is the desired peak current,
I_{mc} is the computed peak current,
$(di/dt)_{maxd}$ is the desired maximum rate of rise of current,
$(di/dt)_{maxc}$ is the computed maximum rate of rise of current,
t_{md} is the desired time to current peak,
t_{mc} is the computed time to current peak,

TABLE I. IMPORTANT WAVE-FRONT PARAMETERS ASSOCIATED WITH LIGHTNING DISCHARGE OF NEGATIVE SS [1].

Current Parameter	Unit	Typical value of SS
Peak value of the current (I_m)	kA	12
Maximum time derivative of the return stroke current $(di/dt)_{max}$	kA/μs	40
Time to crest (t_m)	μs	0.8

TABLE II. HEIDLER'S CBC FUNCTION PARAMETERS FOR "TYPICAL" NEGATIVE SS [13].

I_{01} (kA)	τ_{11} (μs)	τ_{21} (μs)	n_1	I_{02} (kA)	τ_{12} (μs)	τ_{22} (μs)	n_2
10.7	0.25	2.5	2	6.5	2	230	2

As discussed in section II-A, the objective for GA is set to minimize the deviation from the desired value of I_m, $(di/dt)_{max}$ and t_m. The bounds opted for tuning the wave-front of a typical negative SS are reported in Table III. For the case of t_m=1.2 μs the bounds are as specified in brackets. In subsequent sections, the fitness function and bounds opted are appropriately modified, however, the bounds opted, pertaining to this section only are reported (Table III).

Here all the three parameters controlling the wave-front are independently computed, as an extension of the earlier study [19]. The Heidler's function parameters are tuned for different values of t_m (around its typical values of 0.8 μs) by retaining I_m=12 kA and $(di/dt)_{max}$= 40 kA/μs. These

TABLE III. BOUNDS OPTED IN GA FOR HEIDLER'S CBC FUNCTION PARAMETERS IN TUNING THE WAVE-FRONT OF NEGATIVE SS.

GA Variable	Unit	Lower Bound	Upper Bound
I_{01}	kA	10.0	13.0
I_{02}	kA	7.00 (7.50)	9.00 (9.50)
τ_{11}	μs	0 (0.50)	1.00 (1.50)
τ_{21}	μs	0 (1.50)	1.5 (3.00)
τ_{12}	μs	0 (1.50)	2 (3.50)
τ_{22}	μs	210	300
n_1	–	2.00	10.0
n_2	–	2.00	2.50

resultant CBC waves obtained using GA tuned parameters are depicted in figures 2 and 3. This proves the capability of GA in independently tuning CBC function parameters to achieve all the three parameters describing the wave-front.

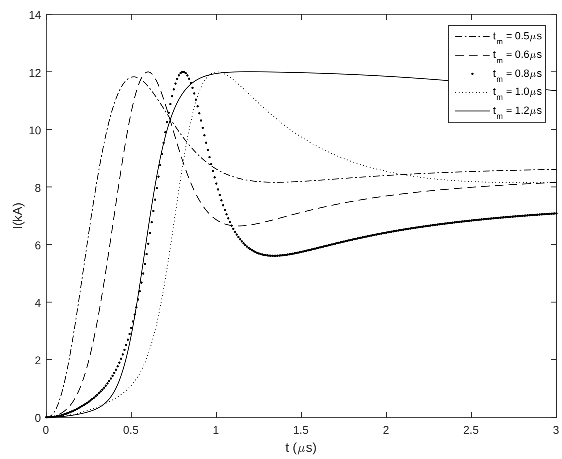

Fig. 2. Plot of GA tuned Hediler's CBC function with t_m as parameter (for a fixed "typical" I_m of 12 kA and $(di/dt)_{max}$ of 40kA/μs) of SS.

D. Tuning of CBC function parameters for negative FS and SS, including those controlling wave-tail

It is reported in literature [7], [9] that a lightning stroke can be almost completely characterized by 5 (or in some cases, 6) parameters, namely, $I_m,\ (di/dt)_{max},\ t_m,\ t_{50},\ (di/dt)_{30-90}$ and Q. From extensive field measurements, the standards concerning lightning protection systems [8] report that these parameters can be closely approximated to follow a log-normal distribution, and provide the median (50%) and severe (5%) values. In this section, the application of GA to compute the CBC parameters, for negative FS and SS, so as to obtain the reported lightning characteristics is discussed.

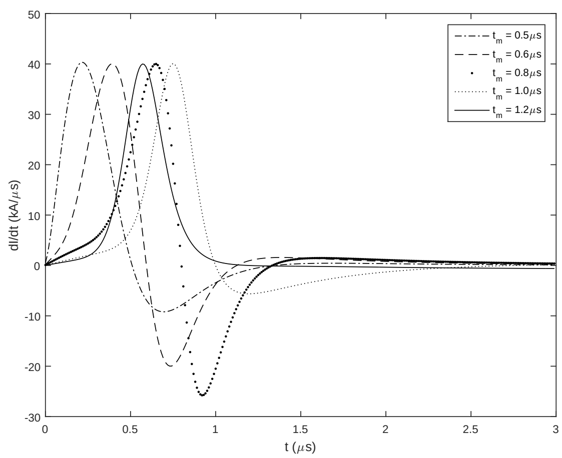

Fig. 3. Plot of time rate of change of Heidler's CBC as a function of time, corresponding to the plots given in figure 2.

In case of negative FS, GA is applied to tune 4 parameters, namely, I_m, $(di/dt)_{max}$, t_m and t_{50}, for both the median and severe case. The fitness function used in this GA application is presented in (3).

$$f = \left(\left|\frac{I_{md} - I_{mc}}{I_{md}}\right|\right) + \left(\left|\frac{(di/dt)_{maxd} - (di/dt)_{maxc}}{(di/dt)_{maxd}}\right|\right) + \left(\left|\frac{t_{md} - t_{mc}}{t_{md}}\right|\right) + \left(\left|\frac{t_{50d} - t_{50c}}{t_{50d}}\right|\right) \quad (3)$$

where,
t_{50d} is the desired time to 50% current peak,
t_{50c} is the computed time to 50% current peak,
and, other parameters are same as those presented in (2)

For negative SS, an additional parameter, the current rate of rise from 30 to 90 percent $(di/dt)_{30-90}$ is also included. The fitness function for this case is presented in (4).

$$f = \left(\left|\frac{I_{md} - I_{mc}}{I_{md}}\right|\right) + \left(\left|\frac{(di/dt)_{maxd} - (di/dt)_{maxc}}{(di/dt)_{maxd}}\right|\right) + \left(\left|\frac{t_{md} - t_{mc}}{t_{md}}\right|\right) + \left(\left|\frac{t_{50d} - t_{50c}}{t_{50d}}\right|\right) + \left(\left|\frac{(di/dt)_{30-90d} - (di/dt)_{30-90c}}{(di/dt)_{30-90d}}\right|\right) \quad (4)$$

$(di/dt)_{30-90d}$ is the desired rate of rise of current from 30 to 90 percent ,
$(di/dt)_{30-90c}$ is the computed rate of rise of current from 30 to 90 percent,
and other parameters are same as that presented in (3)

III. Results

A. Wave-front region of negative SS

Typical SS (12 kA peak, 40 kA/μs maximum time derivative and 0.8 μs time to crest) CBC, for which the Heidler's function parameters exist in the literature [13], is taken as the example to evaluate Heidler's function parameters using GA as a tool. This is to assess the effectiveness of GA as a tool in tuning Heidler's CBC, by comparing with the established results available in the literature (Rachidi et al. [13]).

In continuing with typical cases of SS parameters discussed above, the present study is focused on tuning of Heidler's function parameters by systematically changing the time-to-crest of the CBC (t_m) of SS, by keeping peak (I_m) and maximum time rate of change of CBC $(di/dt)_{max}$ constant. For 5 different t_m, computed parameters of the Heidler's function are as given in Table IV. Five simulation test runs are performed.The best case (out of the 5 test runs of GA with randomized initial population) GA computed parameters, I_{01}, I_{02}, τ_{11}, τ_{21}, τ_{12}, τ_{22}, n_1, and n_2 for negative SS are given in Table IV. The resultant I_m and $(di/dt)_{max}$ waveform variation with t_m as a parameter is depicted in figures 2 and 3.

B. Concerning complete wave shape adjustment of negative FS and SS

For the adjustment of tail end of the lightning return strokes, it is necessary to tune the time to half the peak, t_{50}. This data has been reported in IEC standard 62305-1 [8]. By appropriately modifying the fitness function, it has been demonstrated that the Heidler's function parameters for the CBC can be computed accurately by GA. The GA tuned Heidler's function parameters for negative FS and SS, for both the median and severe cases are summarized in Table V. figures 4 through 7 contain plots of these CBC and their respective time derivatives for four different cases considered (Severe FS, Median FS, Severe SS, Median SS). In this study, the charge transfer (Q) during the lightning stroke has not been included. This is due to the fact that I_m and t_{50} show a high degree of interdependence with parameter Q (for severe cases), making it difficult to tune using sum of two Heidler's function.

Table V summarizes the Heidler's function parameters obtained using GA for the median (50%) and severe (5%) FS and SS reported in the literature [7], [8], [9]. Table VI compares the literature reported and GA-computed (present work) data for median and severe case, for both, negative first and subsequent return stroke obtained using GA tuned Heidler's parameters given in Table V.

C. Tuning of lightning CBC having steep wave front

Recently, Gamerota et al [9] have recommended a set of salient lightning parameters for positive and negative cloud–to–ground lightning discharges. In case of negative median FS, they have used sum of six Heidler's functions to model the lightning CBC. Sum of six Heidler's function has become necessary since the value of $(di/dt)_{max}$ chosen is very high. In the present study, the same desired steepness of CBC (of 100

TABLE IV. GA COMPUTED HEIDLER'S CBC FUNCTION PARAMETERS FOR "TYPICAL" I_m (=12 KA), $(di/dt)_m$ (=40 KA/μS) AND FOR DIFFERENT t_m.

SS CBC parameters estimated using GA											
I_{01} (kA)	I_{02} (kA)	τ_{11} (μs)	τ_{21} (μs)	τ_{12} (μs)	τ_{12} (μs)	n_1	n_2	I_m (kA)	$(di/dt)_m$ (kA/μs)	t_m (μs)	Error (%)
11.855	8.418	0.678	0.189	0.468	288.892	3	2	11.829	40.288	0.510	4.146
12.412	8.099	0.693	0.126	0.703	274.830	6	2	12.000	40.000	0.600	0.003
12.535	7.205	0.829	0.130	0.927	250.568	10	2	11.998	40.038	0.800	0.107
10.144	8.666	0.802	0.719	1.525	298.190	10	2	11.996	40.054	1.000	0.165
10.119	8.307	0.604	2.527	1.783	225.727	8	2	12.003	39.955	1.200	0.139

TABLE V. GA COMPUTED HEIDLER'S FUNCTION PARAMETERS CORRESPONDING TO LIGHTNING FS AND SS, THOSE LISTED IN THE STANDARD [8].

Type of stroke	I_{01} (kA)	I_{02} (kA)	τ_{11} (μs)	τ_{12} (μs)	τ_{21} (μs)	τ_{22} (μs)	n_1 –	n_2 –
50% FS	9.230	10.803	5.452	5.879	96.898	98.807	24	42
5% FS	40.324	49.719	20.016	19.877	245.753	261.835	60	56
50% SS	5.618	6.204	0.241	0.357	40.011	47.056	4	3
5% SS	18.605	10.156	1.801	1.185	177.645	183.602	62	2

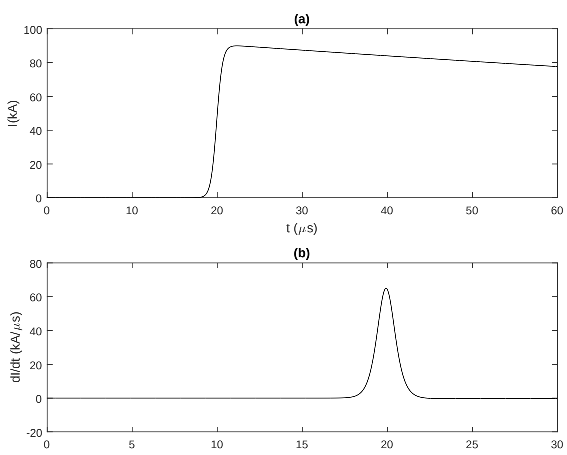

Fig. 4. Severe negative FS (a) current (peak of 90 kA) and (b) time derivative of current (peak of 65 kA/μs) as specified in IEC Standard 62305–1 [8], obtained using GA tuned Heidler's function parameter.

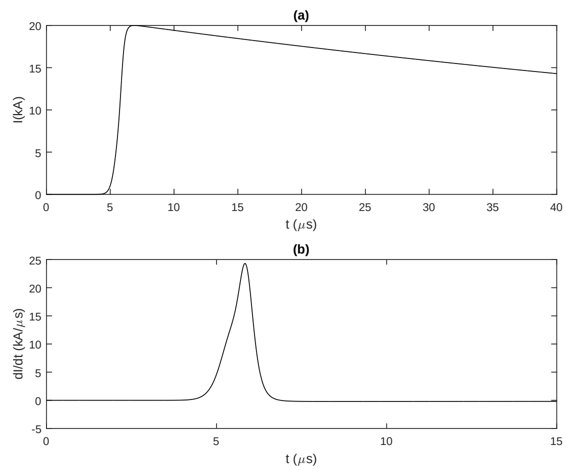

Fig. 5. Median negative FS (a) current (peak of 18 kA) and (b) time derivative of current (peak of 24.3 kA/μs) as specified in IEC Standard 62305–1 [8], obtained using GA tuned Heidler's function parameter.

kA/μs) has been achieved using just two Heidler's functions through the application of GA. The GA tuned Heidler's function parameters corresponding to CBC data (32 kA, 100 kA/μs) of reference [9] are given in Table VII. The computed CBC parameters lie within $\pm 5\%$ of the desired values. A comparison of the lightning CBC, and its time derivative, evolved by GA and those existing literature [9] is presented in Fig. 8.

IV. CONCLUSIONS

GA is successfully applied in computing the lightning CBC Heidler's function parameters ($I_{01}, I_{02}, \tau_{11}, \tau_{21}, \tau_{12}, \tau_{22}, n_1$, and n_2) and to independently adjust all the five parameters of the lightning CBC wave, probably for the first time. It is shown that it is possible to achieve the required CBC wave shape by using GA as the tool with only two Heidler's functions (With consequent reduction in number of parameters). It has also

TABLE VI. COMPARISON OF LIGHTNING PARAMETERS COMPUTED USING GA-TUNED HEIDLER'S CBC WITH IEC STANDARD 62305–1 [8].

Return Stroke Parameter	Measured value (Data from [8])		Modeled value (GA-tuned)		% Error	
	50%	5%	50%	5%	50%	5%
Negative FS						
Peak Current (kA)	20	90	19.996	89.922	0.022	0.087
Time to current peak (10-90 percent) (μs)	5.5	18	5.512	17.848	0.218	0.844
Maximum rate of current rise (kA/μs)	24.3	65	24.300	64.022	0.001	0.344
Time to decay to half peak value (μs)	75	200	74.995	199.080	0.007	0.460
Negative SS						
Peak Current (kA)	11.8	28.6	11.854	28.802	0.455	0.708
Time to current (10-90 percent) peak (μs)	1.1	4.5	1.080	4.496	0.089	1.818
Maximum rate of current rise (kA/μs)	39.9	161.5	38.957	161.518	0.011	2.364
Time to decay to half peak value (μs)	32	140	32.000	133.195	0	4.861
30 to 90 percent rate of current rise (kA/μs)	20.1	98.5	20.100	98.380	0	0.122

TABLE VII. GA COMPUTED HEIDLER'S CBC FUNCTION PARAMETERS CORRESPONDING TO THE STEEP FRONTED (MEDIAN NEGATIVE FS) CBC OF REFERENCE [9].

I_{01} (kA)	I_{01} (kA)	τ_{11} (μs)	τ_{12} (μs)	τ_{21} (μs)	τ_{22} (μs)	n_1 –	n_2 –
16.363	20.210	1.680	2.510	23.924	247.146	38	3

been demonstrated that the wave decay region can also be accurately modeled, in case of both negative FS and negative SS. The worst case errors in the I_m, $(di/dt)_{max}$, t_m, t_{50} and t_{30-90} thus obtained using GA are less than $\pm 4.8\%$. The Heidler's function parameters for median and severe case (FS and SS) described in IEC Standard 62305–1 are evaluated using GA and are reported; which would find their application in lightning research.

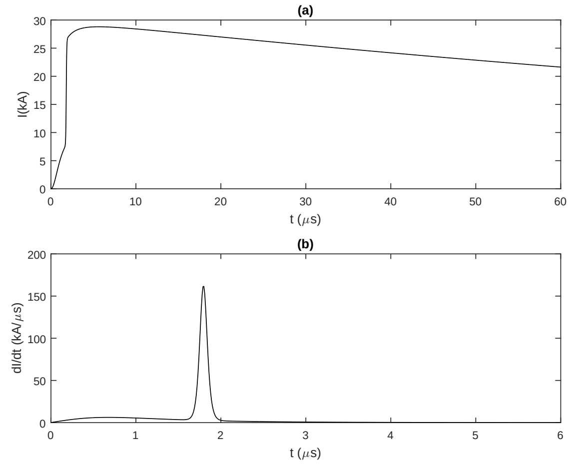

Fig. 6. Severe negative SS (a) current (peak of 28.6 kA) and (b) time derivative of current (peak of 161.5 kA/μs) as specified in IEC Standard 62305–1 [8], obtained using GA tuned Heilder's function parameter.

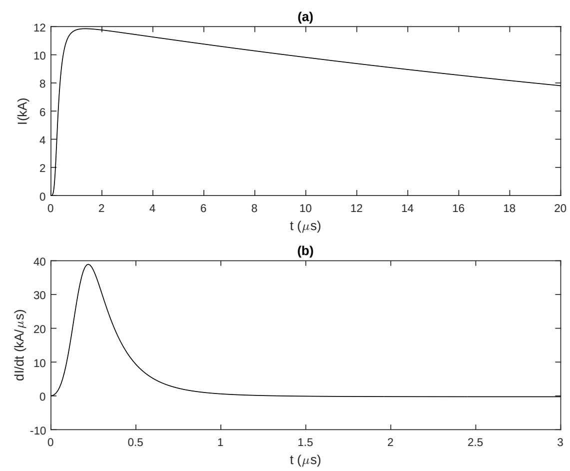

Fig. 7. Median negative SS (a) current (peak of 11.8 kA) and (b) time derivative of current (peak of 39.9 kA/μs) as specified in IEC Standard 62305–1 [8], obtained using GA tuned Heidler's function parameter.

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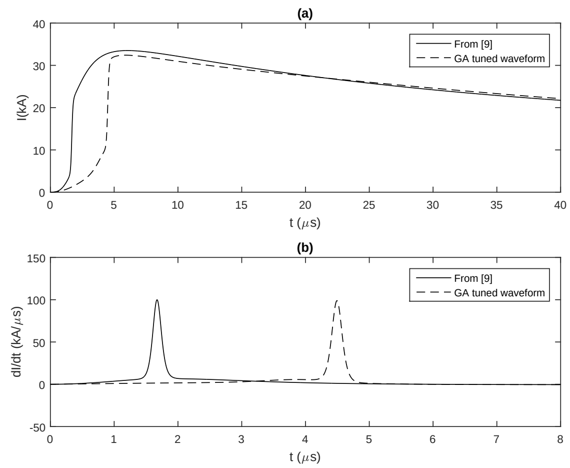

Fig. 8. Median negative FS (a) current (steep wave-front as given in reference [9]) and (b) time derivative of current compared with GA tuned CBC.

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